

# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



INVERSION OF SOME LAPLACE TRANSFORMS  
ENCOUNTERED IN HEAT TRANSFER PROBLEMS

by

John E. Brock,  
Abdollah Zargary,  
and

Craig Comstock

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INVERSION OF SOME LAPLACE TRANSFORMS  
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The body of this monograph presents the statement and proof of a theorem which affords the Laplace inversion of a rather general function encountered in our studies of some conjugated heat transfer problems. Appendices develop some suggestions concerning numerical evaluations and also present additional Laplace inversions which have not been previously tabulated.



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INVERSION OF SOME LAPLACE TRANSFORMS  
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by

JOHN E. BROCK  
Professor of Mechanical Engineering, NPS

ABDOLLAH ZARGARY  
Lieutenant, Imperial Iranian Navy

and

CRAIG COMSTOCK  
Associate Professor of Mathematics, NPS

Introduction

The Laplace inversions treated herein arise in connection with some conjugated heat transfer problems, details of which have been reported in a thesis by one of the authors [4] and will also be reported elsewhere, as well. In connection with another physical problem, R. A. Kantola [1] arrived at similar results. However, the details here are sufficiently involved and sufficiently different than those dealt with by Kantola to suggest preserving our analysis in a monograph such as this.

In the body of this monograph, a theorem is stated and proved which affords the Laplace inversion of a rather general function. Appendix A indicates a suggested numerical procedure for evaluation of the definite integrals appearing in the result. Appendix B treats certain details of a special but useful case. Appendix C merely lists some additional Laplace inversions which have come to our attention in connection with related work



but which do not result from the analysis given in the body hereof.

### Statement of Theorem

The principal content of this monograph is contained in the

THEOREM: Let

$$f(s) = \frac{1}{s} \exp[-\sqrt{m(s)}] \quad (1)$$

where  $m(s)$  is a meromorphic function whose zeros  $s = -a_k$  and whose poles  $s = -b_k$  satisfy

$$0 < a_1 < b_1 < a_2 < b_2 < \dots < a_N < b_N \leq \infty \quad (2)$$

Then, for  $t > 0$ , the inverse Laplace transform  $F(t) = \mathcal{L}^{-1}\{f(s)\}$

is given by

$$F(t) = \exp[-\mu(0)] - \frac{1}{\pi} \sum_{k=1}^N \int_{a_k}^{b_k} \frac{1}{x} \exp(-xt) \sin[\mu(x)] dx \quad (3)$$

where

$$\mu(x) = \sqrt{|m(-x)|} \quad (4)$$

The sum, having upper limit  $N$ , is to be interpreted as follows.

If there is a finite number  $N$  of pairs  $a_k < b_k$ , no special interpretation is necessary. If there is a finite number  $(N-1)$  of pairs  $a_k < b_k$  and an additional zero  $s = -a_N$ ,  $a_N > b_{N-1}$ , an  $N$ th pole  $s = -b_N = -\infty$  is to be added so as to make  $N$  pairs. If there is an infinite number of pairs  $a_k < b_k$ , take  $N = \infty$ .

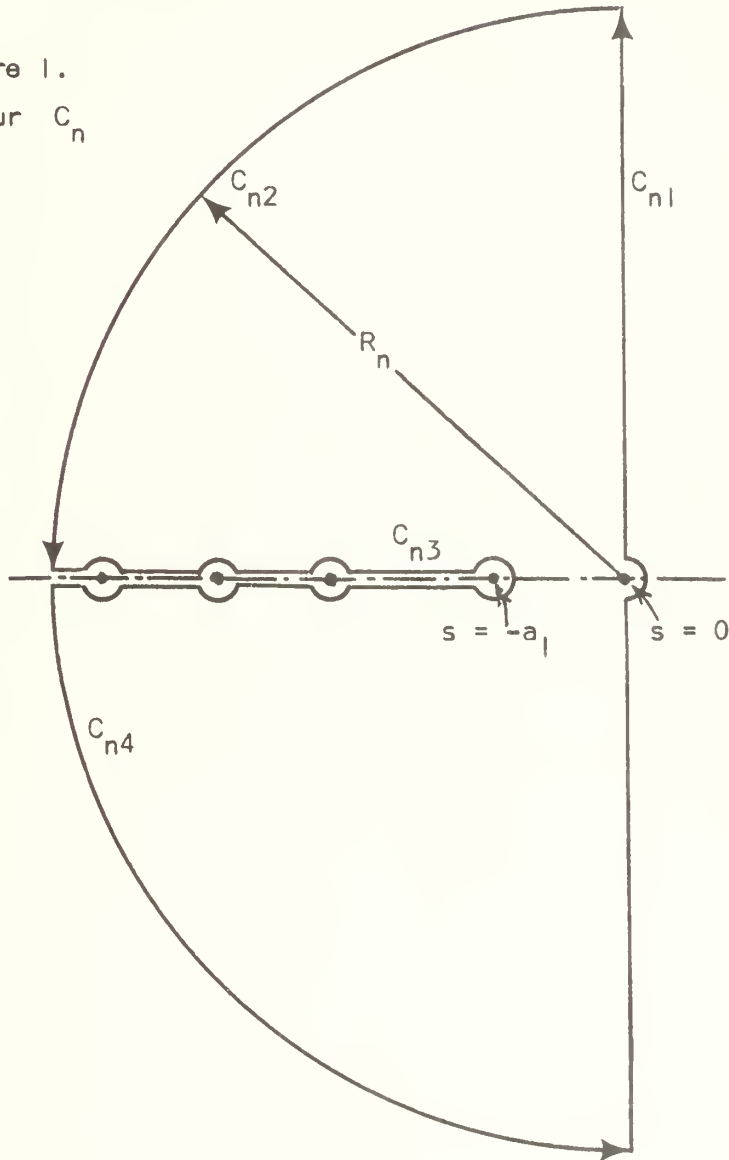
### Proof of Theorem

In the proof we consider a sequence  $\{R_n\}$  of positive numbers,  $R_n \neq a_k, b_k$  for any  $n$  and any  $k$ ,  $R_n \rightarrow \infty$  as  $n \rightarrow \infty$ , and we consider a corresponding sequence of closed contours  $C_n$  in the complex  $s$ -plane each of which encloses the pole of  $f(s)$  at  $s = 0$  but excludes all of the branch



points at  $s = -a_k$  and  $s = -b_k$ . The contour  $C_n$  consists of four sub-contours  $C_{n1}$ ,  $C_{n2}$ ,  $C_{n3}$ , and  $C_{n4}$  as follows.  $C_{n1}$  is a vertical line from  $s = -i R_n$  to  $s = i R_n$ , indented as shown by a small semicircle passing

Figure 1.  
Contour  $C_n$



to the right of  $s = 0$ .  $C_{n2}$  is the arc  $s = R_n e^{i\theta}$ ,  $\frac{\pi}{2} \leq \theta \leq \pi$  and  $C_{n4}$  is the similar arc  $s = R_n e^{i\theta}$ ;  $-\pi \leq \theta \leq -\frac{\pi}{2}$ . Finally,  $C_{n3}$  is the contour which passes to the right from  $s = -R_n$ , along the negative real  $s$ -axis to  $s = -a_1$  with small semicircular indentations into the upper half



s-plane about each zero  $(-a_k)$  and about each pole  $(-b_k)$  after which it then passes to the left from  $s = -a_1$  to  $s = -R_n$  with small semicircular indentations into the lower half s-plane, as indicated in Figure 1.

Clearly, by Cauchy's theorem

$$J_n = \frac{1}{2\pi i} \int_{C_n} e^{st} f(s) ds = \exp [-\sqrt{m(0)}] = \exp [-\mu(0)] \quad (5)$$

for  $n = 1, 2, 3, \dots$ . Also, by the complex inversion theorem for Laplace transforms

$$J_{n1} = \frac{1}{2\pi i} \int_{C_{n1}} e^{st} f(s) ds \rightarrow F(t) \quad (6)$$

as  $n \rightarrow \infty$ .

To proceed further, we wish to establish an inequality for  $\arg[m(s)]$ . To do this, we consider complex vectors emanating from the points  $s = -a_k$  and  $s = -b_k$ , and we write

$$1 + s/a_k = u_k \exp(i\phi_k); u_k \geq 0; -\pi \leq \phi_k \leq \pi \quad (7)$$

$$1 + s/b_k = v_k \exp(i\psi_k); v_k \geq 0; -\pi \leq \psi_k \leq \pi \quad (8)$$

$$m_n(s) = m(0) \prod^{(n)} (1 + s/a_k) \div \prod^{(n)} (1 + s/b_k) \quad (9)$$

$$= \mu_n^2(s) \exp(i\beta_n) \quad (10)$$

where

$$\mu_n^2(s) = \mu(0) \prod^{(n)} u_k \div \prod^{(n)} v_k \quad (11)$$

and

$$\beta_n = \sum \phi_k^{(n)} - \sum \psi_k^{(n)} \quad (12)$$

The sums and products written with a superscript  $(n)$  indicate that only those terms are to be considered for which  $a_n < R_n$  or  $b_n < R_n$ , as may be appropriate.



Now note for any  $s$  in the (closed) upper half plane we have

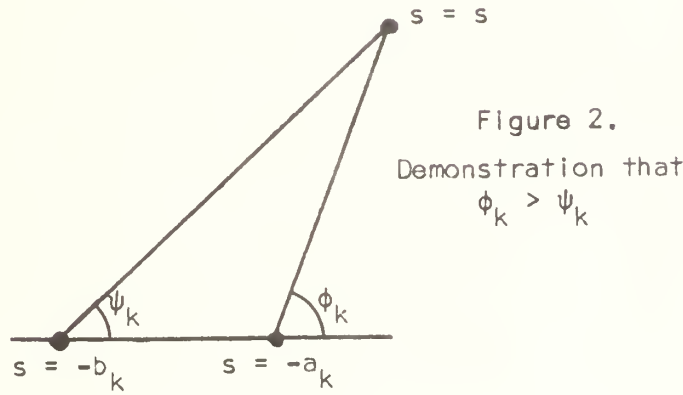
$$\phi_k \geq \psi_k, \quad k = 1, 2, \dots \quad (13)$$

A geometrical argument is given in Figure 2. Now either there are  $N$  pairs  $a_k, b_k$  each less than  $R_n$ , so that

$$\beta_n = \sum_{k=1}^N (\phi_k - \psi_k) \geq 0 \quad (14)$$

or there is an extra, unpaired zero, so that

$$\beta_n = \sum_{k=1}^{N-1} (\phi_k - \psi_k) + \phi_N \geq 0 \quad (15)$$



In the first case, we can write

$$\beta_n = \phi_1 - \sum_{k=1}^{N-1} (\psi_k - \phi_{k+1}) - \psi_N \leq \phi_1 - \psi_N < \phi_1 \leq \pi \quad (16)$$

whereas in the second case we can write

$$\beta_n = \phi_1 - \sum_{k=1}^N (\psi_k - \phi_{k+1}) \leq \phi_1 \leq \pi \quad (17)$$

Thus, in either case,

$$0 \leq \beta_n \leq \pi \quad (18)$$

Similarly, by considering an  $s$  in the lower half plane, we find

$$-\pi \leq \beta_n \leq 0 \quad (19)$$



so that in all cases

$$-\pi \leq \beta_N \leq \pi \quad (20)$$

Thus, if we write

$$J_{n2} = \frac{1}{2\pi i} \int_{C_{n2}} e^{st} f(s) ds \quad (21)$$

we note that

$$\begin{aligned} |f(s)| &= \frac{1}{R_n} \left| \exp(-\sqrt{\mu_n^2 e^{i\beta}}) \right| \\ &= \frac{1}{R_n} \exp[-\mu_n \cos(\beta_n/2)] \leq \frac{1}{R_n} \end{aligned} \quad (22)$$

uniformly on  $C_{n2}$ , by virtue of the inequality (20). Thus, by Jordan's lemma  $J_{n2} \rightarrow 0$  as  $n \rightarrow \infty$ . Similarly, the integral on  $C_{n4}$  vanishes as  $n \rightarrow \infty$ .

Next we consider the integrals around semicircles surrounding poles and zeros on contour  $C_{n3}$ . The result is obvious near a zero:  $s = -a_k$ ;  $e^{st} \approx e^{-a_k t}$ ,  $|f(s)| \approx \frac{1}{a_k}$  since  $m(s) \approx 0$ , and the length of the arc goes to zero as its radius does. Thus, the integral on such a semicircle about a zero,  $s = -a_k$ , vanishes with the radius. However, for the integral on a semicircle about a pole,  $s = -b_k$ , it becomes necessary to use a considerably more subtle argument. For this purpose we must invoke (20) again. We have

$$|\exp[-m(s)]| = \exp[-\mu_n \cos(\beta_n/2)] < 1 \quad (23)$$

even though  $\mu_n \rightarrow \infty$  as the radius of the indentation goes to zero.

Thus, the integral on such a semicircle about a pole,  $s = -b_k$ , vanishes with the radius.



We remark that although we have reason to believe, on physical grounds, the correctness of an evaluation by Kantola [1], we feel that his argument intended to show vanishing of such an integral on a semicircle surrounding a branch point is deficient. Our argument, based on (20), is not applicable to his case.

If now we write (5) in the form

$$J_{n1} = J_n - J_{n2} - J_{n3} - J_{n4} \quad (24)$$

and let  $n \rightarrow \infty$ , and also let the radii of all semicircular indentations on  $C_{n3}$  go to zero, we find

$$F(t) = \exp[-\mu(0)] - J_3 \quad (25)$$

where

$$J_3 = \frac{1}{2\pi i} \int_{C_3} \frac{1}{s} \exp[st - \sqrt{m(s)}] ds \quad (26)$$

and  $C_3$  now is a contour extending from  $s = -\infty$  along the upper edge of a branch cut, to the right, passing around  $s = -a_1$ , and then extending along the lower edge of the branch cut to  $s = -\infty$ . The branch cut is the negative real  $s$ -axis from  $-\infty$  to  $-a_1$ . There are no longer any semicircular indentations in  $C_3$ .

To reduce (26) to a more convenient form, we first introduce the function  $v(x)$ , for real  $x$ , which denotes the excess of the number of zeros ( $s = -a_k$ ) over the number of poles ( $s = -b_k$ ) to the right of the point  $s = x$ . It is clear that  $v = 0$  in the interval  $-a_{k+1} < s < -b_k$  while  $v = 1$  in the interval  $-b_k < s < -a_k$ . Writing  $\beta$  now, rather than  $\beta_n$ , since we have taken  $n \rightarrow \infty$  and all zeros and poles are included in our accounting, we note that on the upper edge of the cut  $\beta = v\pi/2$



while on the lower edge  $\beta = -v\pi/2$ . We also now write  $\mu$  rather than  $\mu_n$ .

Thus,

$$\begin{aligned}
 J_3 &= \frac{1}{2\pi i} \int_{-\infty}^{-a_1} \frac{1}{s} \exp[st - \mu \exp(iv\pi/2)] ds \\
 &+ \frac{1}{2\pi i} \int_{-a_1}^{-\infty} \frac{1}{s} \exp[st - \mu \exp(-iv\pi/2)] ds \\
 &= \frac{1}{2\pi i} \int_{-\infty}^{-a_1} \frac{1}{s} e^{st} \left\{ \exp[-\mu \exp(iv\pi/2)] - \exp[-\mu \exp(-iv\pi/2)] \right\} ds
 \end{aligned} \tag{27}$$

If  $v = 0$ , the quantity in braces vanishes, while if  $v = 1$ , it becomes

$$\exp(-i\mu) - \exp(i\mu) = -2i \sin \mu \tag{28}$$

Thus

$$J_3 = -\frac{1}{\pi} \sum_{k=1}^N \int_{-b_k}^{-a_k} \frac{1}{s} e^{st} \sin \mu \, ds \tag{29}$$

If we now change the variable of integration from  $s$  to  $x = -s$ ,

we obtain

$$J_3 = \frac{1}{\pi} \sum_{k=1}^N \int_{a_k}^{b_k} \frac{1}{x} e^{-xt} \sin[\mu(x)] dx \tag{30}$$

and, thus, finally

$$F(t) = \exp[-\mu(0)] - \frac{1}{\pi} \sum_{k=1}^N \int_{a_k}^{b_k} \frac{1}{x} e^{-xt} \sin[\mu(x)] dx$$

The use of the upper limit of summation is consistent with the remarks

following the statement of the theorem, the proof of which is now complete.



## REFERENCES

- (1) Kantola, R. A., Transient Response of Fluid Lines Including Frequency Modulated Inputs, (Ph.D.) Thesis, Polytechnic Institute of Brooklyn, 1969 (University Microfilms, Inc., Ann Arbor, Michigan.)
- (2) Kopal, Z., Numerical Analysis, John Wiley and Sons, Inc., 1955.
- (3) Roberts, G. E., and Kaufman, H., Table of Laplace Transforms, W. B. Saunders Company, 1966.
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## APPENDIX A

### Numerical Evaluation of the Integrals

The integrals appearing in the inversion theorem (3) are of the form

$$I = \int_a^b h(x) \sin[g(x)] dx \quad (A-1)$$

where  $h(x)$  is continuous in  $[a, b]$  (A-2)

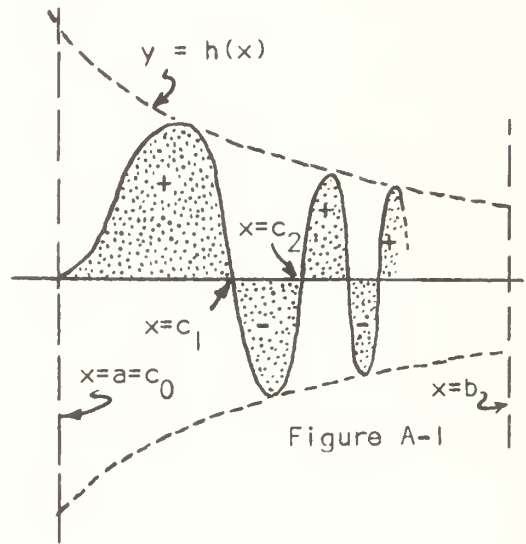
$g(x)$  is monotonic continuous in  $[a, b]$  (A-3)

$g(a) = 0$  (A-4)

$g(x) \rightarrow \infty$  as  $x \rightarrow b^-$  (A-5)

It is contemplated that  $b$  may be finite or infinite.

The integrand is illustrated in the figure at the right. The integral,  $I$ , is the algebraic sum of the shaded areas, with signs taken as indicated. For values of  $x$  sufficiently close to  $b$ , these contributions will alternate in sign so that  $I$  may be regarded as the sum of an alternating series of terms having decreasing magnitude.



Thus, the error of truncation is less than the magnitude of the first term to be neglected.

In the numerical treatment one must evaluate, in succession, the numbers  $C_n$  such that

$$g(C_n) = n\pi \quad (A-6)$$

The values  $C_n$  evidently satisfy

$$a = C_0 < C_1 < C_2 < \dots < b \quad (A-7)$$



This determination is uniquely possible because the monotonicity of  $g(x)$  assures the existence of a unique inverse function. Many techniques are available for this evaluation. Some remarks suggesting a procedure suited to the problems which form the subject of this monograph will appear near the end of the present appendix.

Thus, we have

$$I = \lim_{N \rightarrow \infty} \sum_{k=1}^N q_k \quad (A-8)$$

where

$$q_k = \int_{C_{k-1}}^{C_k} h(x) \sin[g(x)] dx \quad (A-9)$$

Numerical quadrature is indicated. A formula of Gaussian type might be appropriate except that the ordinary Gaussian points are all interior to the interval of integration and thus cannot reflect our knowledge that  $\sin[g(C_n)] = 0$  for  $n = 0, 1, 2, \dots$ . A slightly more efficient procedure, accordingly, employs Gauss-Radau integration in which the end points of the interval are specifically included among the integration points. The integrand is zero at these points so that the corresponding summands may be omitted. For details of Gauss-Radau formulas, see Z. Kopal [2]. The appropriate formula is

$$\int_{-1}^1 F(x) dx = H_1 F(-1) + \sum_{j=2}^{N-1} H_j F(a_j) + H_N F(1) \quad (A-10)$$

where  $a_j = -a_{n-j+1}$  and  $H_j = H_{n-j+1}$  are tabulated in Appendix 4.5 of Kopal [2] for  $N = 2, 3, \dots, 11$ . (Note that these  $a_j$ 's should not be confused with  $a_k$ 's which locate the zeros of the function  $m(s)$  employed in the body of this monograph).



For example, for  $n = 5$ , we have

GAUSS-RADAU CONSTANTS FOR $N = 5$	
$a_5 = 1 = -a_1$	$H_5 = H_1 = 1/10$
$a_4 = \sqrt{3/7} = -a_2$	$H_4 = H_2 = 49/90$
$a_3 = 0$	$H_3 = 64/90$

We have calculated the coefficients  $a_j$  and  $H_j$  for the case  $N = 12$  and list the values here so as to supplement the information given in [2].

GAUSS-RADAU CONSTANTS FOR $N = 12$	
$a_{12} = 1.00000\ 00000\ 00000\ 0$	$H_{12} = 0.01515\ 15151\ 51515\ 2$
$a_{11} = 0.94489\ 92722\ 22882\ 0$	$H_{11} = 0.09168\ 45174\ 13204\ 7$
$a_{10} = 0.81927\ 93216\ 44007\ 4$	$H_{10} = 0.15797\ 47055\ 64367\ 1$
$a_9 = 0.63287\ 61530\ 31861\ 8$	$H_9 = 0.21250\ 84177\ 61024\ 6$
$a_8 = 0.39953\ 09409\ 65348\ 9$	$H_8 = 0.25127\ 56031\ 99201\ 1$
$a_7 = 0.13655\ 29328\ 54927\ 6$	$H_7 = 0.27140\ 52409\ 10696\ 2$

These formulas lead to exact evaluation for a polynomial  $F(x)$  of degree  $2N-3$ . In our case, with appropriate mapping of  $[C_n, C_{n+1}]$  onto  $[-1, 1]$ , we have  $F(-1) = F(1) = 0$  so that using  $N = 9$ , say, requires seven nontrivial evaluations, leading to exact integration of a 15-degree polynomial, whereas with ordinary Gaussian integration, we would have had to use  $N = 8$ , i.e., eight nontrivial evaluations for exact integration of a 15-degree polynomial.



The actual evaluation of  $q_k$  thus becomes

$$q_k = \frac{1}{2} (C_k - C_{k-1}) \sum_{j=2}^{N-1} H_j \cdot h(x_j) \sin [q(x_j)] \quad (A-11)$$

where

$$x_j = \frac{1}{2} \left[ (C_k + C_{k-1}) + a_j (C_k - C_{k-1}) \right] \quad (A-12)$$

The value of  $N$  to be used should be consistent with the accuracy required and the costs of computation

To test this procedure, we considered the special case where

$$h(x) = (1/\pi - x)^{-2} \exp[-1/(1/\pi - x)] \quad (A-13)$$

$$q(x) = \pi x / (1/\pi - x) \quad (A-14)$$

$$a = 0; b = 1/\pi \quad (A-15)$$

We easily find

$$C_n = n/\pi(n+1) \quad (A-16)$$

and using  $N = 6$ , we obtain the following evaluation

$\underline{k}$	$c_k$	$q_k$	$\Sigma q_k$
0	0	-	-
1	$1/2\pi$	.0225407	.0225407
2	$2/3\pi$	-.0009741	.0215666
3	$3/4\pi$	.0000421	.0216087
4	$4/5\pi$	-.0000018	.0216069
5	$5/6\pi$	.0000001	.0216070

In this particular case, the integral can be evaluated exactly. We

have

$$I = \pi^2 \int_0^{1/\pi} (1-\pi x)^{-2} \exp[-\pi/(1-\pi x)] \sin [\pi^2 x / (1-\pi x)] dx$$



Let  $1 - \pi x = \pi y$ , and we find

$$\begin{aligned} I &= \int_0^{1/\pi} y^{-2} \exp(-1/y) \sin(1/y - \pi) dy \\ &= - \int_0^{1/\pi} y^{-2} \exp(-1/y) \sin(1/y) dy \end{aligned}$$

Now let  $z = 1/y$ , and we have

$$I = - \int_{\pi}^{\infty} e^{-z} \sin z \, dz = 1/2e^{\pi} = .0216070$$

and the adequacy of the evaluation is evident.

We now consider effective means of evaluating the numbers  $C_n$  which satisfy A-7 and will be bunched more and more closely together as  $n \rightarrow \infty$ . It may be anticipated that difficulties may be encountered when using customary methods of root evaluation. In the applications with which this monograph is concerned,  $g(x) \rightarrow \infty$  as  $x \rightarrow b^-$  because of the factor  $(1 - x/b)^{-1/2}$ . If we assume that in the interval  $(a, b)$  we have

$$[g(x)]^2 = \frac{x-a}{b-x} m(x)$$

where  $m(x)$  is "approximately constant," we are led to the iterative formula

$$C_n^{j+1} = \frac{bn\pi (C_n^j - a) + a(b - C_n^j) [g(C_n^j)]^2}{n\pi (C_n^j - a) + (b - C_n^j) [g(C_n^j)]^2} \rightarrow C_n$$

where  $C_n^j$  is the  $j$ th iterate for  $C_n$ . One may conveniently take

$$C_n^1 = C_{n-1}^{\infty} = C_{n-1}$$

and

$$C_1^1 = \frac{1}{2} (a + b)$$

However, we have not tested this procedure by actual computations.

For the case of two poles and one zero, explicit evaluations for  $C_n$  are easily obtained, as is shown in Appendix B hereof.



## APPENDIX B

### The Case of Two Zeros and One Pole

Consider the special case where

$$m^2(s) = k^2(s+a_1)(s+a_2)/(s+b_1) \quad (B-1)$$

where  $k^2 > 0$  and  $0 < a_1 < b_1 < a_2$ . We have

$$\mu(0) = \mu_0 = k\sqrt{a_1 a_2 / b_1} \quad (B-2)$$

and the inversion is

$$F(t) = e^{-\mu_0 t} - \frac{1}{\pi} \int_{a_1}^{b_1} \frac{1}{x} e^{-xt} \sin \mu \, dx - \frac{1}{\pi} \int_{a_2}^{\infty} \frac{1}{x} e^{-xt} \sin \mu \, dx \quad (B-3)$$

In this case we are able to obtain an explicit representation of the values  $C_n$  introduced in Appendix A. In both regions of integration we have

$$\mu(x) = k\sqrt{(x-a_1)(a_2-x)/(b_1-x)} \quad (B-4)$$

so that the condition

$$\mu(C_n) = n\pi \quad (B-5)$$

(cf. equation (A-6)) leads to

$$C_n = \frac{1}{2} \left[ (a_1 + a_2 + \gamma_n) \pm \sqrt{(a_1 + a_2 + \gamma_n)^2 - 4(a_1 a_2 + b_1 \gamma_n)} \right] \quad (B-6)$$

where

$$\gamma_n = n^2 \pi^2 / k \quad (B-7)$$

The lower sign (-) is to be used for the first integral (from  $a_1$  to  $b_1$ ) and the upper sign (+) is to be used for the second integral (from  $a_2$  to  $\infty$ ).



## APPENDIX C

### Some Additional Inversions

We have also obtained the following inversions which are not related to those considered in the body hereof and which are not listed in Reference [3]. They are reported here for record purposes.

$f(s)$	$F(t)$
$\frac{\cosh\sqrt{as}}{(s-c)\sqrt{s}\sinh\sqrt{bs}}$	$\frac{e^{ct}\cosh\sqrt{ac}}{\sqrt{c}\sinh\sqrt{bc}} - \frac{2}{\sqrt{b}} \sum_{n=0}^{\infty} \frac{(-1)^n \cos(n\pi\sqrt{a/b})}{c + n^2\pi^2/b} e^{-n^2\pi^2 t/b}$
$s^{-n} e^{a/s}$	$(t/a)^{(n-1)/2} I_{n-1}(2\sqrt{at}) \quad (n \geq 1)$

Additionally, we note a rather obvious erratum in 7.2.1.2, p. 284, of Reference [3] where the factor  $-2s$  preceding the sign of summation should be  $-2\pi$ .



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John E. Brock, Professor of Mechanical Engineering, NPS

Abdollah Zargary, Lieutenant, Imperial Iranian Navy

Craig Comstock, Associate Professor of Mathematics, NPS

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13. ABSTRACT

The body of this monograph presents the statement and proof of a theorem which affords the Laplace inversion of a rather general function encountered in our studies of some conjugated heat transfer problems. Appendices develop some suggestions concerning numerical evaluations and also present additional Laplace inversions which have not been previously tabulated.



KEY WORDS	LINK A		LINK B		LINK C	
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